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14. ABSTRACT Experiments are described in which the quantum states of two flux qubits were read out by a dc SQUID (Superconducting QUantum Interference Device). Observations of the linewidths of microwave-induced peaks, Ramsey fringes and flux echoes on a single qubit yielded values of the decoherence times T_2' , T_2^* and T_2 , respectively. Within the experimental errors, the values satisfied the prediction $1/T_2^* = 1/T_2' + 1/T_2$. Spectroscopic measurements on two qubits revealed energy gaps at their degeneracy points matched to within $\pm 1\%$: 8.88 ± 0.01 GHz and 8.98 ± 0.01 GHz, respectively. Detailed measurements were made of the entangled states of the qubits in two regimes: at the double degeneracy point (both qubits biased at a half-flux quantum $\Phi_0/2$), and at flux values away from $\Phi_0/2$ at which two qubit frequencies coincided ("intersecting anticrossing"). The energy levels were determined from microwave transitions from the ground state $ 0\rangle$ to the first and second excited states $ 1\rangle$ and $ 2\rangle$. At the double degeneracy point, a repulsion of the two-energy levels was observed while at the intersecting anticrossing, an avoided crossing was observed. The measured energies were in excellent agreement with calculations with no fitted parameters. Furthermore, the energy splitting at the intersecting anticrossing could be controlled by means of a current bias pulse in the readout SQUID, as predicted theoretically.					
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I. Introduction

The major goals of this project were to design, fabricate and operate two flux qubits, to demonstrate that they could be entangled, and to vary their interaction energy. These goals have been realized and the results are summarized in this final report.

Section II presents a brief overview of the design and fabrication of the two qubits and the dc SQUID (Superconducting QUantum Interference Device) used to measure their quantum states. The infrastructure and measurement techniques are briefly summarized. Section III summarizes measurements of the dephasing times of a single flux qubit using Ramsey fringes, flux echoes and spectroscopic linewidths. These results are used to show that the dephasing is due to $1/f$ flux noise. In Sec. IV, we outline the theory of a scheme to control the interaction between two flux qubits. Recent experimental results on the controlled entanglement of two flux qubits are presented in Sec. V; these results were obtained during the six-month, no-cost extension of the grant that terminated November 30, 2005. Publications and presentations resulting from this research are listed in Appendix A. Appendix B lists the personnel involved in the project.

II. Design Issues and Infrastructure

Our experiments are based on the three-junction flux qubit first proposed by Orlando *et al.* [1], but in a significantly different regime. In the original design, the area of the superconducting loop was very small (dimensions of a few micrometers) so that the inductance of the qubit was entirely dominated by the kinetic inductance of the junctions. The small area of the loop makes the device less sensitive to external magnetic field fluctuations than large loops, but, by the same token, one has to apply a relatively large field to induce a flux on the order of a flux quantum, $\Phi_0 = h/2e \approx 2.07 \times 10^{-15}$ Wb. As a consequence, the Delft group used an off-chip coil to generate the magnetic field. This approach makes it virtually impossible to control the fluxes in two qubits on the same chip independently. Since we wished to have full control of these fluxes, we adopted a different approach. We designed the qubits with areas two orders of magnitude larger than in the Delft design, and equipped each with its own on-chip flux lines capable of producing a flux quantum in the SQUID with a modest current (~ 500 μ A). In this design, the qubit inductance is comparable with the geometrical inductance of the loop, yielding quite different dynamics. As a result, we undertook a detailed study of the qubit behavior in this regime [2]. An obvious drawback of the larger loop area is that the design is correspondingly more sensitive to external magnetic field fluctuations. We obviated this problem by enclosing the chip in a superconducting box that effectively excluded ambient magnetic noise.

We fabricated our devices on oxidized Si chips using electron-beam lithography and double-angle evaporation to form the Al-AlOx-Al tunnel junctions. A photograph of the two qubits and the readout SQUID surrounding them appears in Fig. 1(a). The Al lines for the qubit and SQUID loops are 1 μ m wide, and for the flux bias 10 μ m wide. The Josephson junctions shown in Figs. 1(b) and (c) are approximately 200×200 nm². The layout for the chip is shown in Fig. 2

Figure 3 shows schematically the configuration of the measurement system. The dilution refrigerator is surrounded by a shielded room made of copper sheet with an attenuation of 100 dB at 1 GHz. The refrigerator, an Oxford Kelvinox 300, is capable of attaining temperatures below 20 mK; however, the thermal load of the various wires and cables limits the base temperature to 35–40 mK. The chip on which the superconducting circuits are fabricated is mounted in a copper box, plated on the inside with lead and thermally anchored to the mixing chamber. The pulse and sense lines for the readout SQUID enter from the right-hand cavity, and are heavily damped by 3-k Ω resistors. The two flux bias lines, one for the qubit and the other for the readout SQUID, enter from the left and are heavily damped by copper powder filters in the cavity. Microwaves can be coupled to the qubit via a coaxial line that enters from below and is coupled to the qubit via a 1-mm-diameter loop of niobium wire.

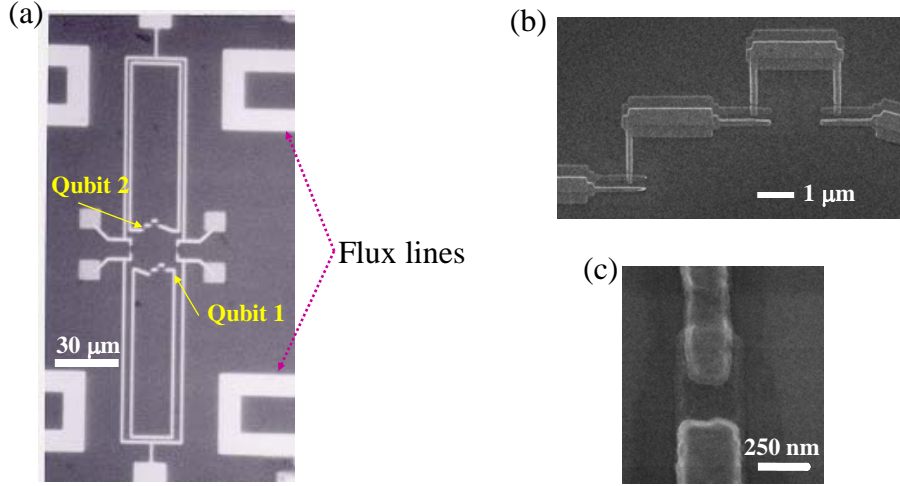


Fig. 1 (a) Photograph of two qubits surrounded by the readout SQUID. Segments of flux lines are visible to the left and right of the SQUID. (b) Scanning electron micrograph of the three junctions of one of the qubits. (c) Scanning electron micrograph of one of the SQUID junctions.

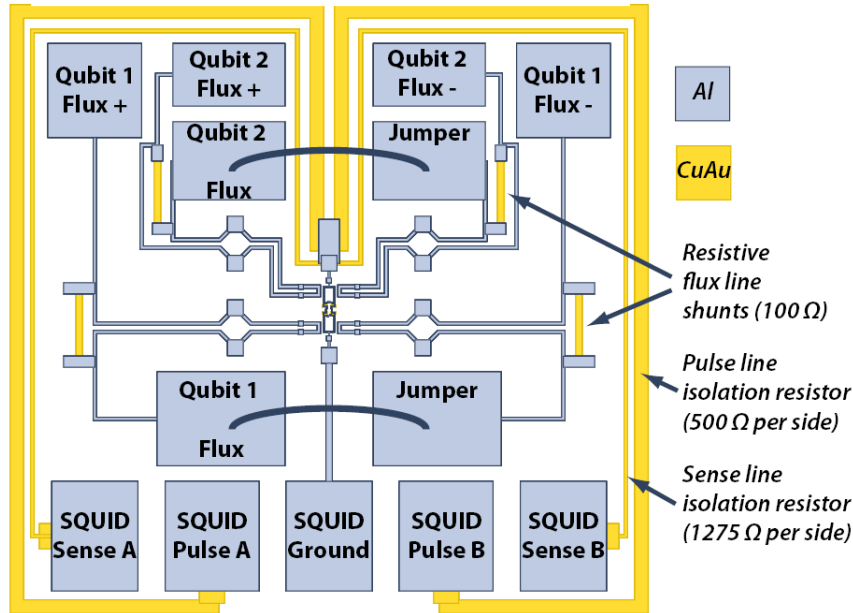


Fig. 2 Chip layout. Blue represents Al traces, gold AuCu traces. Pads near upper edge of chip provide two independent flux lines; Al wirebonded jumpers couple left and right halves of each of the two flux lines. Pads near lower edge of chip are used to supply current pulses to the read-out SQUID and to sense any resulting voltage. Chip is 2 mm on a side.

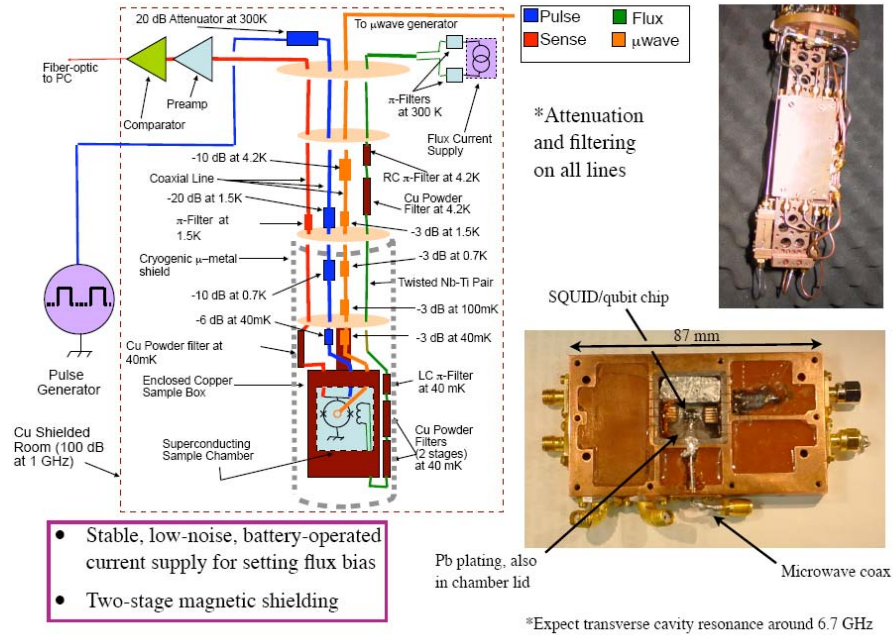


Fig. 3 Dilution refrigerator. Left-hand figure is a schematic of the wiring scheme. Right-hand figures show the box in which the sample is mounted

Figure 3 also shows the various levels of filters and attenuators. To determine the critical current of the readout SQUID, a current pulse is supplied by a computer-controlled generator outside the shielded room. This line has a 20-dB attenuator at room temperature, a 20-dB attenuator at 1.5 K, a 10-dB attenuator at 0.7 K and a 6-dB attenuator at 40 mK. The pulse is largely reflected by the 3-k Ω terminating resistor, but the reflected pulse is absorbed by the matched 50- Ω attenuator at 40 mK so that there is no resonance. The sense line is coupled to the readout SQUID via a 3-k Ω resistor, and the signal passes through a copper powder filter at 40 mK and a π -filter at 1.5 K before being coupled to the room-temperature, battery-operated preamplifier. If the SQUID switches in response to the pulse, the comparator transmits a signal to the computer outside the shielded room via a fiber-optic link. The flux bias currents are produced by digitally programmable potentiometers controlled by a computer via a fiber optic link [3]. The lines to the chip are very heavily filtered: there are π -filters at 300 K and 4.2 K, a copper powder filter at 4.2 K, and a π -filter and two copper powder filters at 40 mK. This series of filters roll off sharply at frequencies above 25 Hz. Finally, the microwave generator, which is outside the shielded room, is inductively coupled to the qubit via a line with the following attenuators: 20 dB at room temperature, 10 dB at 4.2 K, 3 dB at 1.5 K, 3 dB at 100 mK and 3 dB at 40 mK. In addition, there is a high pass filter at 10 MHz. The mutual inductance of the line to the qubit is extremely small, approximately 10 fH.

III. Results on a Single Flux Qubit

A. Measurements of dephasing times

To perform experiments on the entanglement of two qubits, it is essential to be able to manipulate the state of a single qubit by means of microwave pulses in a precise way. We briefly describe results on Ramsey fringes and flux echoes [4] and show that the two values of the decoherence times obtained from those are compatible with the measured linewidth.

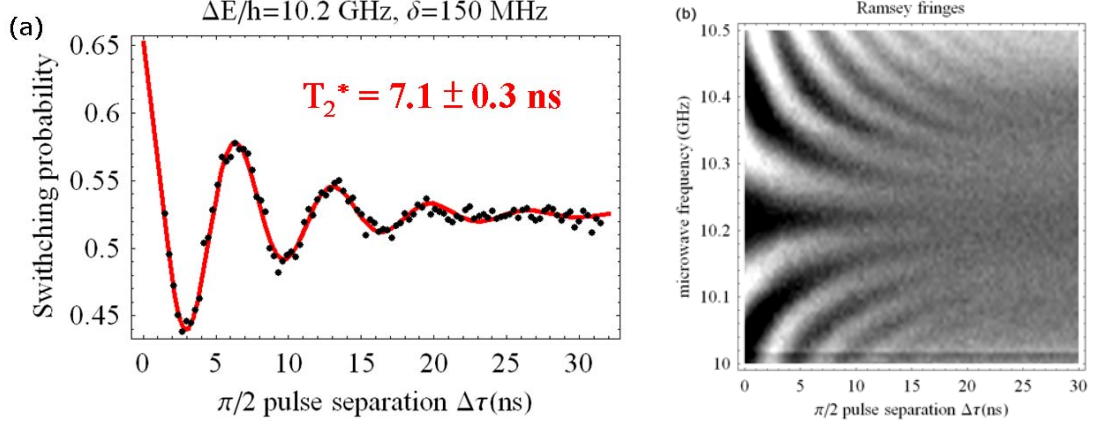


Fig. 4 Ramsey fringes. (a) Ramsey fringe at 150 MHz detuning. (b) Ramsey fringe measurement containing 80 excitation frequencies.

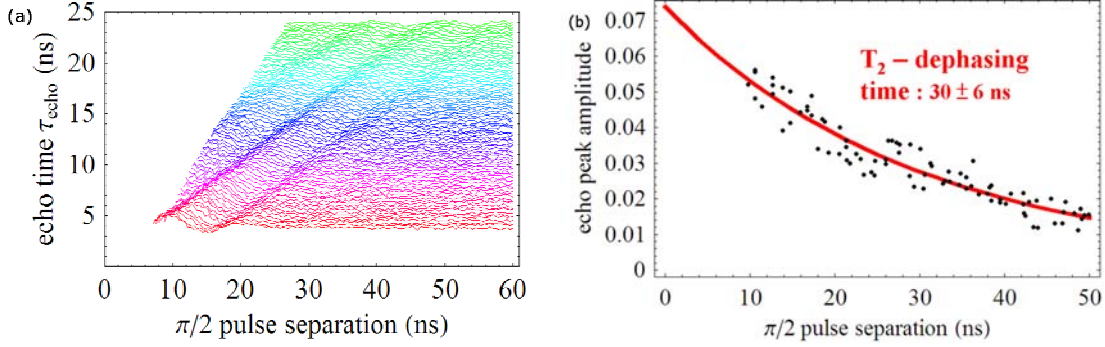


Fig. 5 Flux echoes. (a) Echo-corrected Ramsey fringes. (b) Decay of echo envelope.

These experiments were performed on a flux qubit with a minimum energy splitting (at the degeneracy point with a flux bias of $\Phi_0/2$) of 3.99 GHz. All three measurements were made away from the degeneracy point at a frequency of 10.2 GHz. Figure 4(a) plots the SQUID switching probability versus the time interval between the two pulses for a microwave frequency f_m that is slightly off-resonance. A fit to these oscillations yields a decoherence time T_2^* of 7.1 ± 0.3 ns. The Ramsey fringe frequency is expected to be proportional to $|f_m - f_q|$, where f_q is the resonant frequency of the qubit. This behavior is clearly demonstrated in Fig. 4(b), which yields $f_q = 10.22$ GHz.

We performed echo-corrected Ramsey fringe measurements in which a microwave π pulse was inserted between the two $\pi/2$ pulses. Data for a range of π -pulse positions are shown in Fig. 5(a). The echo pulse corrects for inhomogeneous dephasing, so that the peak of the echo envelope decays with the homogeneous dephasing time T_2 . In Fig. 5(b) we obtain a value $T_2 = 30 \pm 6$ ns.

Another measure of decoherence may be obtained from the dependence of spectroscopic linewidth on microwave amplitude. In the limit of strong driving, this dependence is approximately linear and extrapolates at zero driving amplitude to a linewidth corresponding to the inhomogeneous dephasing time T_2' . Figure 6 shows linewidth measurements at a qubit resonant frequency of 10.2 GHz, and the extrapolation of $T_2' = 11.1 \pm 1$ ns from these data.

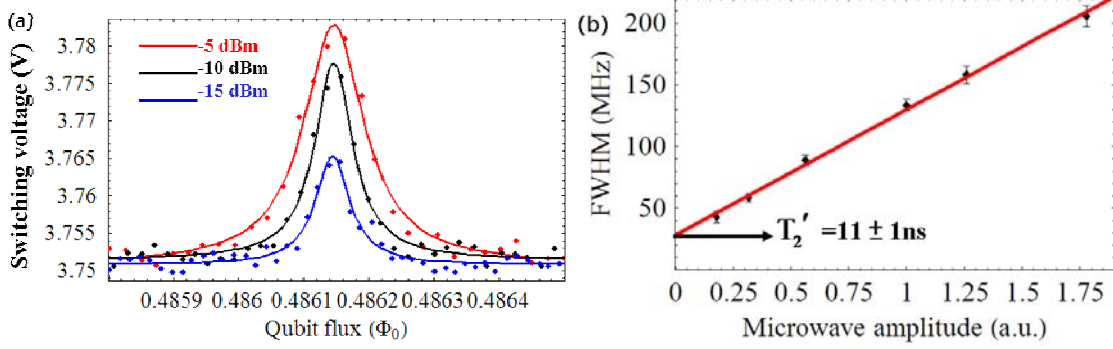


Fig. 6 Linewidth dependence on microwave amplitude. (a) Spectroscopic peaks at varying microwave amplitude. Excitation frequency was 10.2 GHz. (b) Peak linewidth as a function of microwave amplitude.

The overall dephasing rate is expected to be the sum of the homogeneous and inhomogeneous contributions, that is, $(T_2^*)^{-1} = (T_2)^{-1} + (T_2')^{-1}$. We find $[(T_2)^{-1} + (T_2')^{-1}]^{-1} = 8.1 \pm 1.7$ ns, which is consistent with the measured value $T_2^* = 7.1 \pm 0.3$ ns.

B. Estimates of flux noise

We have investigated $1/f$ flux noise as a possible source of dephasing in our qubit. A given flux noise spectrum S_Φ produces dephasing in a Ramsey-fringe measurement according to

$$\langle(\Delta\phi)^2\rangle_{\text{Ramsey}} = 2 (2I_p/\hbar)^2 \int_{f_m}^{\infty} S_\Phi(f) [\sin(\pi f t)/\pi f]^2 df, \quad (1)$$

where $\langle(\Delta\phi)^2\rangle$ is the phase variance and f_m is the inverse of the measurement time. The dephasing time is the time required for the dephasing factor $\exp(-\langle(\Delta\phi)^2\rangle/2)$ to decay to $1/e$. Dephasing in an echo-corrected Ramsey-fringe measurement may be calculated similarly, from

$$\langle(\Delta\phi)^2\rangle_{\text{echo}} = 2 (2I_p/\hbar)^2 \int_{f_m}^{\infty} S_\Phi(f) [\sin^2(\pi f t/2) / (\pi f/2)]^2 df. \quad (2)$$

We assume a noise spectrum of the form $S_\Phi(f) = \alpha^2 / f^\beta$, where $\alpha = S_\Phi^{1/2}(f = 1 \text{ Hz}) \times 1 \text{ Hz}$, and set $f_m = 4 \text{ Hz}$. We compute numerically the values of α and β consistent with our measured dephasing times. Equating the two expressions [Eqs. (1) and (2)] for $\langle(\Delta\phi)^2\rangle$ we find $\beta \approx 0.95$ and a noise amplitude of about $4.5 \mu\Phi_0$ at 1 Hz. Thus, these measurements yield a flux noise power spectrum that scales closely as $1/f$. This measurement was made possible by our determination of T_2 and T_2^* on the same qubit under precisely the same experimental conditions.

It is extremely interesting to compare the inferred value of $1/f$ noise with the $1/f$ measurements of Wellstood *et al.* [5] on 12 dc SQUIDs, made almost 20 years ago. The SQUIDs were fabricated from Nb, Pb, or PbIn in a variety of configurations. These data were obtained by measuring the $1/f$ voltage noise across a voltage-biased SQUID as a function of the applied flux using a second SQUID as an amplifier. The variation of the noise with the flux-to-current transfer function enabled the authors to separate out the flux $1/f$ noise from the critical current $1/f$ noise. The magnitude of the noise at the lowest temperatures varied from about 5 to $10 \mu\Phi_0$ at 1 Hz. The remarkable agreement of these values with the value deduced from the recent flux qubit measurements is quite striking, and leads one to postulate the existence of a “universal flux $1/f$ noise” to join the “universal critical current $1/f$ noise” [6] and “universal charge $1/f$ noise” that pervade the superconducting community. Elucidating the fundamental mechanisms for these noise sources remains a challenging and vital problem.

IV. Entangling Flux Qubits with a Bipolar Dynamic Inductance

To implement a quantum algorithm, one must be able to entangle multiple qubits, so that an interaction term is required in the Hamiltonian describing any two qubit system. For two superconducting flux qubits, the natural interaction is between their magnetic fluxes. Thus, for two flux qubits arranged so that a flux change in one qubit can alter the flux of the second qubit, the coupled-qubit Hamiltonian takes the form

$$H = H_1 + H_2 - (K/2) \sigma_z^{(1)} \sigma_z^{(2)}. \quad (1)$$

Here, H_1 and H_2 are the Hamiltonians of the separated qubits 1 and 2, and $\sigma_z^{(1)}$ and $\sigma_z^{(2)}$ are the Pauli spin-1/2 operators for qubits 1 and 2. The parameter K characterizes the coupling strength; for $K < 0$, the minimum energy configuration for this system is with the qubits antiparallel. For two qubits coupled through a mutual inductance M_{qq} , the interaction strength K_0 is given by

$$K_0 = -2M_{qq} |I_q^{(1)}| |I_q^{(2)}|, \quad (2)$$

and will always be present in any practical layout; $I_q^{(1)}$ and $I_q^{(2)}$ are the supercurrents in qubits 1 and 2.

Entangling operations can be much more efficient if the interaction can be varied and, ideally, turned off during parts of the manipulation. We have devised a coupling scheme for flux qubits whereby the interaction is adjusted by changing a relatively small current which can be varied rapidly with existing pulse technology [7]. For suitable device parameters, the sign of the coupling can also be changed, thus making it possible to null out the direct interaction between the flux qubits. Furthermore, the same device can be used to vary the coupling and to read out the flux states of the qubits. A variable coupling scheme for charge-based superconducting qubits has been suggested recently [8], but, to our knowledge, no feasible variable coupling has previously been proposed to exploit the advantages of flux qubits.

The coupling mechanism for our scheme [Fig. 7(a)] is the circulating current J in a dc Superconducting QUantum Interference Device (SQUID), in the zero voltage state, which is coupled to each of the two identical qubits through an identical mutual inductance M_{qs} . A variation in the flux applied to the SQUID, Φ_s , caused by one of the qubits switching between its two flux states, changes J as determined by $L_D^{-1} = \partial J / \partial \Phi_s$, the inverse dynamic inductance. This change in J alters the flux coupled from the SQUID to the second qubit, leading to an interaction energy between the two qubits, K_s , which is proportional to L_D^{-1} . The coupling strength K_s takes the form

$$K_s = -2M_{qs}^2 |I_q^{(1)}| |I_q^{(2)}| \text{Re}(\partial J / \partial \Phi_s). \quad (3)$$

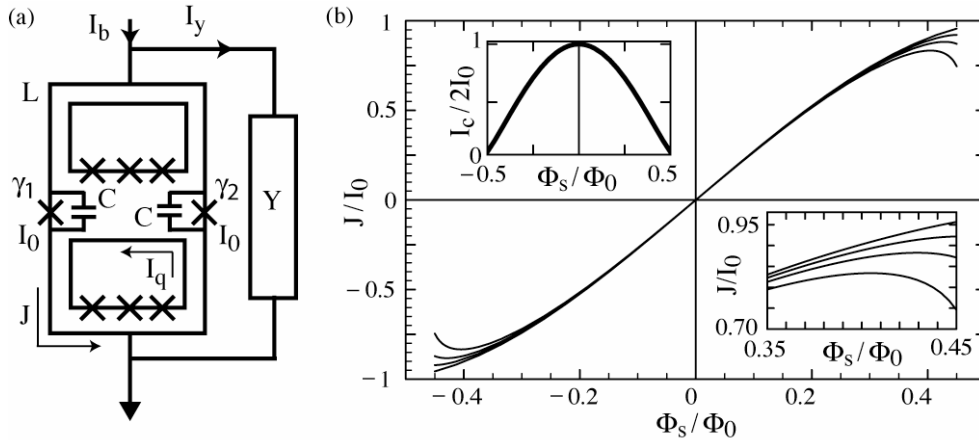


Fig. 7 (a) Layout of SQUID-based coupling scheme. Element Y represents the admittance of the SQUID bias and measurement circuitry. (b) Response of SQUID circulating current J to applied flux Φ_s for $I_b/I_c(0.45 \Phi_0) = 0, 0.4, 0.6, 0.85$ and $\beta_L = 0.092$. Inset in lower right shows variation of J with I_b near $\Phi_s = 0.45 \Phi_0$ and upper left inset shows modulation of I_c with Φ_s .

For reasonable device parameters, the sum of K_s and the direct mutual interaction between the qubits, K_0 , can be as large as -0.3 GHz. For a particular range of values of the screening parameter $\beta_L = 2LI_0/\Phi_0$ and for fixed magnetic flux, L_D^{-1} can be reduced to zero and even made negative by changing the bias current I_b [Fig. 7(b)]; here $I_b < I_c(\Phi_s)$, the critical current for which the SQUID switches out of the zero-voltage state at zero temperature in the absence of quantum tunneling. The ability to change the sign of L_D^{-1} makes it possible to null out K_0 , yielding a vanishing net interaction so that the qubits are truly isolated [Fig. 8]. In single qubit experiments, the qubit state is determined by coupling the qubit screening flux to a similar dc SQUID to which rapid pulses of I_b are applied to measure $I_c(\Phi_s, T)$. Thus, the existing measurement technology allows for I_b to be varied on this same timescale of a few nanoseconds, so that a single dc SQUID can be used to measure the flux state of two qubits and to couple them together controllably.

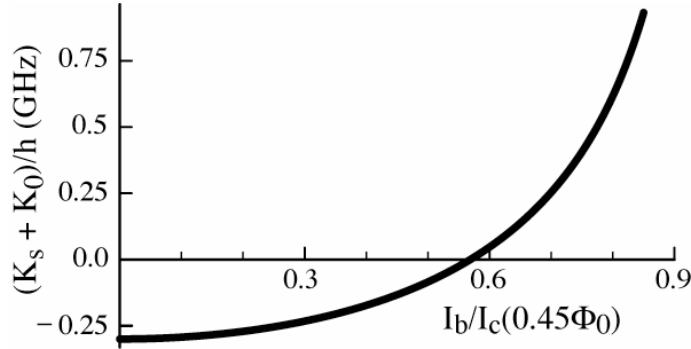


Fig. 8 Variation of K with I_b for $\Phi_s = 0.45 \Phi_0$ and device parameters given in text.

V. Two Entangled Flux Qubits

A. Characterization of individual qubits

Subsequently, we fabricated a new chip and succeeded in entangling the two qubits. Figure 9 shows the microwave spectroscopy of the two qubits. Remarkably, the two devices have nearly identical parameters: at the degeneracy point, the splittings were $\Delta_A = 8.88 \pm 0.01$ GHz and $\Delta_B = 8.98 \pm 0.01$ GHz, respectively. The fact that the splittings agree to within $\pm 1\%$ indicates that the areas and critical currents of the junctions are very closely matched.

In contrast to our earlier flux qubits, it proved possible to operate each qubit separately at its degeneracy point. At the degeneracy points, the values of T_2 obtained from echoes were 74 ± 11 ns and 148 ± 12 ns for qubits A and B, respectively; away from the degeneracy points, the corresponding times were 23 ± 4 and 43 ± 4 ns. The longer values of T_2 at the degeneracy points suggest that dephasing is due to flux noise, which to first order has no effect there.

B. Measurements on entangled qubits

There are two fundamentally different modes in which two qubits can be entangled. The first occurs when both qubits are flux biased away from their degeneracy points. As is evident from Fig. 9, in the absence of any qubit-qubit interaction, the two spectra would intersect. The presence of the interaction, however, resolves the degeneracy and causes an anticrossing: we refer to this as an “intersecting anticrossing”. The magnitude of the splitting is related to the magnitude of the qubit-qubit interaction. The second kind of entanglement occurs when both qubits are biased very close to their respective degeneracy points: this is possible because of the independent flux lines. The interaction between the two qubits shifts the values of Δ_A and Δ_B . We refer to this kind of entanglement as occurring at the double degeneracy point.

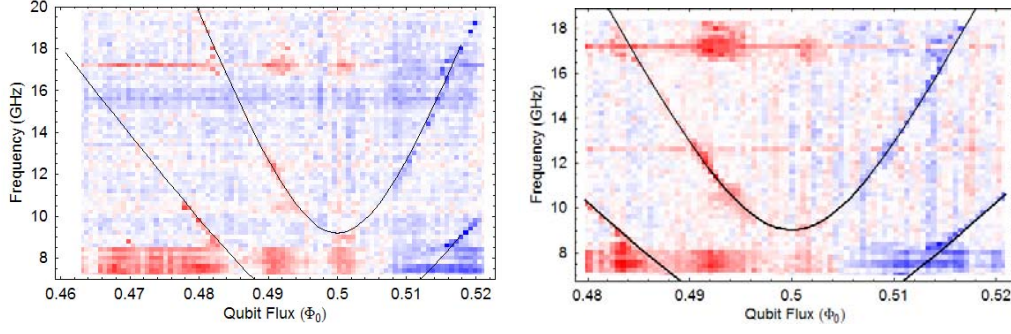


Fig. 9 Microwave spectroscopy of two flux qubits. (a) qubit A, (b) qubit B.

The basis states for the two qubits are $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. Here “0” refers to the spin-up state of one of the qubits and “1” to the spin-down state. The qubit-qubit interaction leads to the formation of four new states, three symmetric triplet states of the form $|00\rangle$, $\alpha|01\rangle + \beta|10\rangle$ and $|11\rangle$ and one antisymmetric singlet state of the form $\gamma|01\rangle - \delta|10\rangle$. The direct interaction between the two qubits is antiferromagnetic, that is, the state $|01\rangle$ or $|10\rangle$ has a lower energy than the state $|00\rangle$ or $|11\rangle$.

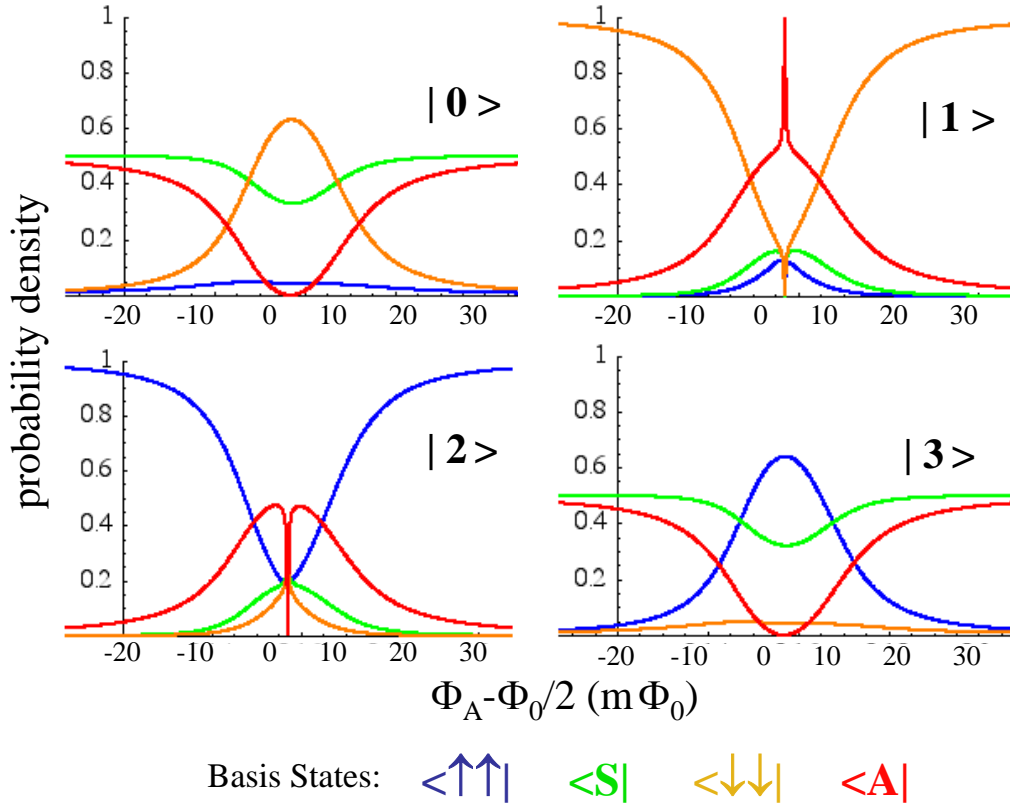


Fig. 10 Calculated wave functions near the intersecting degeneracy point. Probability density for each of the four basis states is plotted for the states $|0\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$.

Precisely at the intersecting anticrossing (Fig. 10) for two identical qubits, the ground state $|0\rangle$ is a triplet, the first excited state $|1\rangle$ is a singlet, the second excited state $|2\rangle$ is a triplet and the third excited state $|3\rangle$ is a triplet. For the small differences in Δ_A and Δ_B relevant to our experiment, there will be tiny amounts of the singlet state mixed in with the triplet states and *vice versa*. Furthermore, as one changes the

flux biases away from the degeneracy points, there will also be small changes in the contributions of the four basis states. Figure 11(a) shows spectroscopy near an intersecting anticrossing at 11.24 GHz. The upper spectrum is the triplet state $|2\rangle$ and the lower curve the singlet state $|1\rangle$. These data were obtained by adjusting the bias currents in the two flux lines to sweep the fluxes in the two qubits Φ_A and Φ_B , while keeping the flux in the SQUID constant. The observed splitting at the anticrossing is 122.6 ± 0.8 MHz. It is noteworthy that there is a paucity of data for the singlet state near the anticrossing. The lack of transitions from the ground state is as predicted: the perturbation of the form $\sigma_{1Z} + \sigma_{2Z}$ is symmetry-preserving so that transitions from a pure triplet state to a pure singlet state are forbidden. As the flux biases move away from the degeneracy point, there is a small admixture of singlet state into the ground state and of triplet state into the excited state $|1\rangle$, thereby permitting transitions. The corresponding squares of the matrix elements calculated for the transitions to the states $|2\rangle$ and $|1\rangle$, $|T_{20}|^2$ and $|T_{10}|^2$, are plotted in Fig. 12(a) for the measured parameters of our system. We see that, while $|T_{20}|^2$ varies smoothly across the degeneracy point, $|T_{10}|^2$ drops to zero at the degeneracy point. The behavior is entirely consistent with the data.

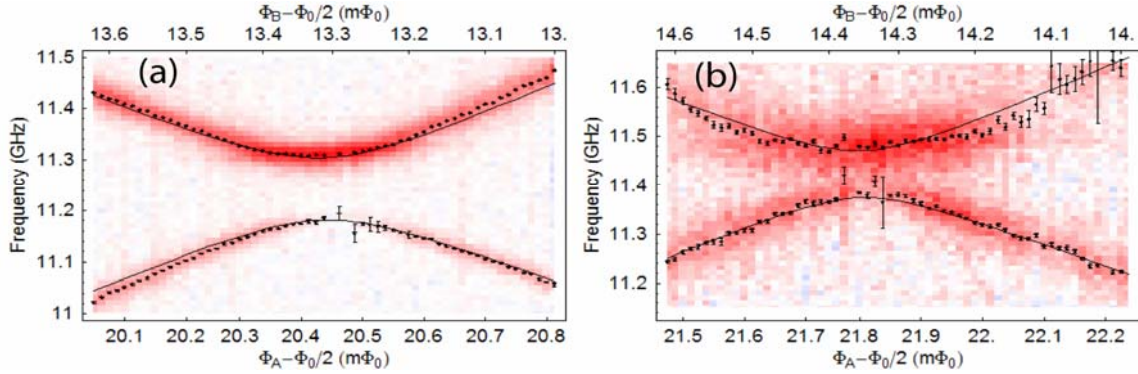


Fig. 11 Microwave spectroscopy for entangled qubits A and B near an intersecting anticrossing. (a) Zero SQUID bias current. The upper state is the triplet excited state $|2\rangle$, the lower the singlet excited state $|1\rangle$. Note the absence of transitions to $|1\rangle$ close to the degeneracy point, as predicted by theory. The splitting at the degeneracy point is 122.6 ± 0.8 MHz. (b) Spectroscopy in the presence of a 0.46- μ A bias current in the readout SQUID. The splitting at the degeneracy point has been reduced to 96.7 ± 2.6 MHz. In both figures, the fluxes Φ_A and Φ_B were adjusted so as to keep the total flux in the SQUID constant.

Figure 12(b) shows the calculated splitting of the $|0\rangle$ and $|2\rangle$ states. These calculations involve the following parameters: the self-inductance of the SQUID and the mutual inductances between the two qubits and between each qubit and the SQUID (estimated from Fast Henry); the maximum critical current of the SQUID (calculated from its measured resistance well above the energy gap); the measured flux bias of the SQUID; and the values of the qubit circulating supercurrents, determined from the asymptotic slopes of the spectra in Fig. 9 ($d\varepsilon/d\Phi = 2I_q$, where ε is the frequency in energy units). The calculated value of the splitting at the degeneracy point is 119 MHz. The extraordinarily good agreement with the measured value of 122.6 ± 0.8 MHz is probably coincidental, since the estimated values of the inductances are likely to be accurate to no more than a few percent. Nonetheless, this result demonstrates that the coupling of the two qubits is well understood despite the substantial nonlinear contribution of the SQUID.

Figure 11(b) shows the spectroscopy in the presence of a small bias current pulse applied to the SQUID just before a larger current pulse is applied to make a measurement of the state of the qubits. The splitting at the degeneracy point has been reduced to 96.7 ± 2.6 MHz. As explained in Sec. IV, the bias current modifies the dynamic inductance of the readout SQUID, thereby reducing the magnitude of the coupling energy. In turn, this reduces the splitting. At the time of writing, a systematic study is being made of the dependence of the splitting on SQUID bias current, and the results will be compared with the theory of Sec. IV.

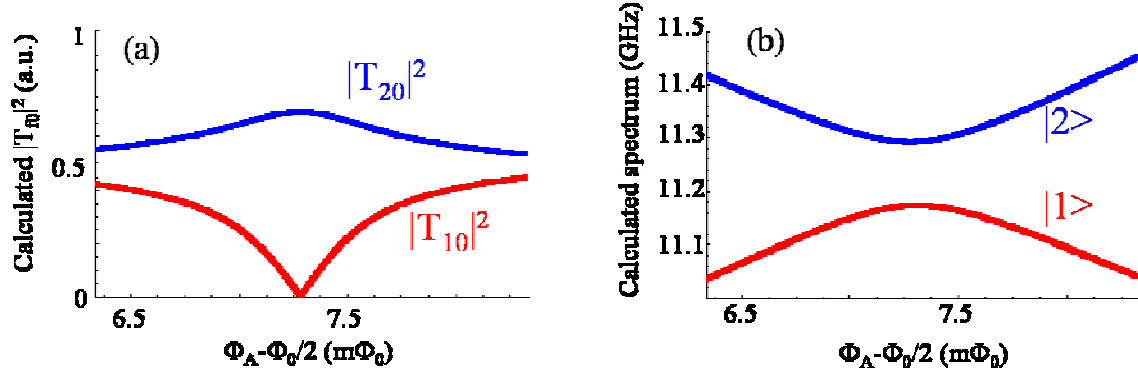


Fig. 12 Calculated behavior at the intersecting degeneracy point for the parameters of Fig. 11(a). (a) Matrix elements $|T_{10}|^2$ and $|T_{20}|^2$. (b) Spectrum of the states $|1\rangle$ and $|2\rangle$

We turn now to a discussion of the qubit-qubit interaction near the double degeneracy point. In this case, the ground state $|0\rangle$ and third excited state $|3\rangle$ are again pure triplet states (Fig. 13). The first excited state is predominantly singlet, but has an admixture of the triplet states; by the same token, the second excited state is predominantly triplet, but has an admixture of the singlet state. Figure 14 shows the measured spectra near the double degeneracy point at which both qubits are biased near $\Phi_0/2$. The solid lines are the measured spectra for the uncoupled qubits. We see immediately that the interaction between the qubits has repulsed the energy levels of the two coupled states $|1\rangle$ and $|2\rangle$. The state $|2\rangle$ has moved up by 55 ± 7 MHz and the state $|1\rangle$ has moved down by 53 ± 7 MHz. It is noteworthy that, given the experimental errors, the repulsion of these two energy levels by the qubit-qubit coupling, 108 ± 7 MHz, is close to that measured at the intersecting degeneracy point.

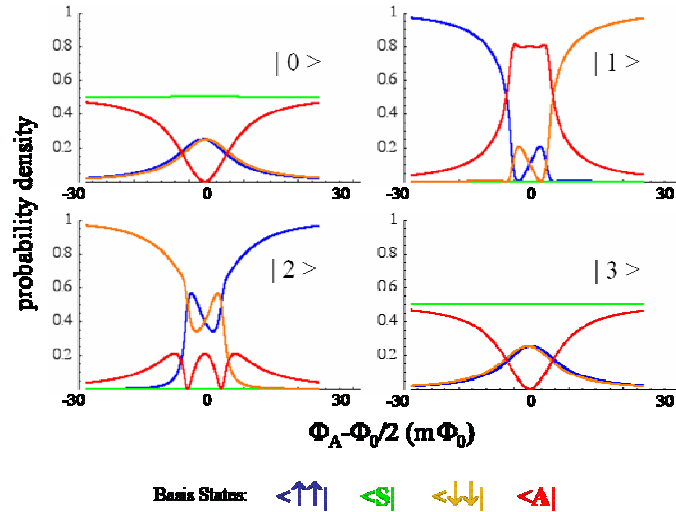


Fig. 13 Calculated wave functions near the double degeneracy point. Probability density for each of the four basis states is plotted for the states $|0\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$.

We expect that experiments will continue for several months to explore many more aspects of these coupled qubits. These experiments include controlling the energy level splitting at both the intersecting anticrossing and the double degeneracy point by varying the bias flux and bias current of the SQUID, performing various kinds of manipulations – Rabi oscillations, Ramsey fringes and flux echoes – on the entangled states, and attempting a Rabi swap, in which one first populates (say) the $|2\rangle$ state and applies a pulse to transfer this population to (say) the state $|1\rangle$.

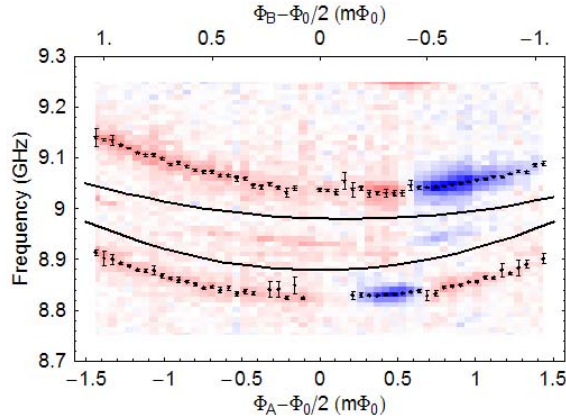


Fig. 14 Microwave spectroscopy for entangled qubits A and B near their double degeneracy point. Upper state is $|2\rangle$ and lower state is $|0\rangle$. The solid lines are the spectra for the uncoupled qubits, lower is A and upper is B. Coupling the qubits repulsed the two levels by a total of 108 ± 7 MHz.

VI. Concluding remarks

These experiments demonstrate the feasibility of entangling two flux qubits in two fundamentally different modes: at the intersecting anticrossing and at the double degeneracy point. This entanglement involves the quantum superpositions of two macroscopic states containing about 10^{12} electrons. Microwave spectroscopy reveals the resultant anticrossing in the first case and energy level shift in the second. Application of a bias current pulse in the SQUID in the zero-voltage state changed the magnitude of the splitting in each case, by an amount that was in accord with predictions. In principle, the two qubits and their readout SQUID should be capable of a Controlled-NOT (CNOT) operation.

Given the successful observation of the controlled entanglement of two flux qubits, what steps could we take to improve the performance? In fact, the on-chip devices are sufficiently close to optimum that it is unlikely that we could improve on them significantly without a substantially better electron-beam writing system. Thus, any improvements would be directed at the infrastructure and readout scheme, and are as follows.

(i) Replace the dissipative readout scheme with a nondissipative scheme. Currently, the rate at which we can acquire data is limited to a few kilohertz by the fact that each SQUID switching event injects heat into the chip that takes hundreds of microseconds to decay. A better scheme would determine the flux applied to the SQUID by the qubits by measuring its inductance, leaving the SQUID in the zero-voltage state and thus causing no dissipation. Two methods of achieving this goal have been reported. One, by the Delft group, involves connecting the SQUID in parallel with a capacitor and measuring the resonant frequency of the resulting tank circuit. The resonant frequency depends on the flux in the SQUID. The second method, invented by Devoret and co-workers at Yale, involves a Josephson bifurcation amplifier. Both methods have the advantage of employing relatively narrow band signals to interrogate the SQUID, thereby potentially reducing decoherence due to low-frequency noise being coupled to the qubits. Furthermore, the absence of dissipation implies that one can greatly increase the rate of data acquisition and thus considerably reduce the time to perform a given experiment.

(ii) Replace the microwave coupling loop with on-chip microwave lines. Careful design of these lines should reduce the microwave coupling to the readout SQUID substantially, and thereby minimize the disturbance of the SQUID by microwave pulses used to manipulate the flux qubits. Furthermore, it would be desirable to install two microwave lines, one for each qubit, to enable one to manipulate the state of the qubits independently.

(iii) Attempt to increase the dephasing times. This is a challenging and open-ended undertaking. If indeed the decoherence times are limited by a “universal flux noise” of unknown origin, the way forward is not obvious. Nonetheless, we could attempt to improve the environment provided by the various lines coupled to the chip by improving the microwave engineering. In particular, we could probably reduce the low-frequency noise coupled in via the flux lines by means of improved on-chip filters. Noise coupled down the line used to apply readout signals to the SQUIDs could be reduced by means of a low pass filter at about 1 GHz. However, implementation of an inductive readout scheme will likely reduce this source of noise in any case. To the extent we can make reasonable estimates, noise coupled in via the microwave line and SQUID readout line is not important. Whether or not these suggested improvements will enhance the dephasing times remains to be seen, but they seem to be the only way to attack the problem.

To make further progress towards a quantum computer, one would of course have to implement large numbers of qubits. Fabrication of (say) tens of qubits will be challenging, but, given good facilities, seems perfectly feasible. Before attempting such proliferation, however, it would seem wise to explore the limits of what can be achieved in terms of dephasing times and proficiency of readout schemes with just two qubits. Subsequent scaling would likely be much more straightforward.

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Appendix A: Publications and Conference Presentations

Publications

1. Flux Qubit Completes the Hat Trick, John Clarke, *Science* **299**, 1850 (2003).
2. Vortices and Hearts, John Clarke, *Nature News and Views* **425**, 133 (2003).
3. Low-Noise Computer-Controlled Current Source for Quantum Coherence Experiments, S. Linzen, T.L. Robertson, T. Hime, B.L.T. Plourde, P.A. Reichardt, and John Clarke, *Rev. Sci. Instrum.* **75**, 2541 (2004).
4. Decoherence in Josephson-junction Qubits due to Critical Current Fluctuations, D.J. Van Harlingen, T.L. Robertson, B.L. T. Plourde, P.A. Reichardt, T.A. Crane, and John Clarke, *Phys. Rev. B* **70**, 064517 (2004).
5. Entangling Flux Qubits with a Bipolar Dynamic Inductance, B.L.T. Plourde, J. Zhang, K.B. Whaley, F.K. Wilhelm, T.L. Robertson, T. Hime, S. Linzen, P.A. Reichardt, C.-E. Wu and John Clarke, *Phys. Rev. B* **70**, 140501 (2004).

6. Measurements of $1/f$ Noise in Josephson Junctions at Zero Voltage: Implications for Decoherence in Superconducting Quantum Bits, Michael Mück, Matthias Korn, C.G.A. Mugford, J.B. Kycia and John Clarke, *Appl. Phys. Lett.* **86**, 012510 (2005).
7. Superconducting Quantum Interference Device with Frequency-Dependent Damping: Readout of Flux Qubits, T.L. Robertson, B.L.T. Plourde, T. Hime, S. Linzen, P.A. Reichardt, F.K. Wilhelm and John Clarke, *Phys. Rev. B* **72**, 024513 (2005).
8. Flux Qubits and Readout Device with Two Independent Flux Lines, B.L.T. Plourde, T.L. Robertson, P.A. Reichardt, T. Hime, S. Linzen, C.-E. Wu and John Clarke, *Phys. Rev. B* **72**, 060506 (2005).
9. Quantum Theory of Three-Junction Flux Qubit with Non-Negligible Loop Inductance: Towards Scalability, T.L. Robertson, B.L.T. Plourde, P.A. Reichardt, T. Hime, C.-E. Wu and John Clarke, *Phys. Rev. B*, to be published.

Conference Presentations

1. "Critical Current $1/f$ Noise in Josephson Junctions: Implications for Flux Qubits," John Clarke, AFOSR Review, Cambridge, MA, July, 2002
2. "RC-shunted SQUIDs for Single-shot Measurement of Flux Qubits," invited talk, B.L.T. Plourde, T.L. Robertson, P.A. Reichardt, T. Hime, D.J. Van Harlingen, J. Clarke, Applied Superconductivity Conference, Houston, TX, August 9, 2002.
3. "Characterization and implications of low frequency noise in superconducting phase qubits," invited talk, D.J. Van Harlingen, B.L.T. Plourde, T.L. Robertson, P.A. Reichardt, R. Therrien, J. Clarke, LT 23, Hiroshima, Japan, August, 2002.
4. "Decoherence in Flux Qubits due to $1/f$ Noise," T.L. Robertson, D.J. Van Harlingen, B.L.T. Plourde, P.A. Reichardt and J. Clarke, March Meeting of the American Physical Society, Austin, Texas, March 3-7, 2003.
5. "RC-Shunted SQUIDs for Single-Shot Measurement of Flux Qubits," B.L.T. Plourde, T.L. Robertson, T. Hime, S. Linzen, P.A. Reichardt, J. Clarke and D.J. Van Harlingen, March Meeting of the American Physical Society, Austin, Texas, March 3-7, 2003.
6. "Critical Current $1/f$ Noise in Josephson Junctions: Implications for Flux Qubits," Tim Robertson, Britton Plourde, Paul Reichardt, Dale Van Harlingen and John Clarke, invited talk by John Clarke at Wilhelm und Else Heraeus-Stiftung Seminar, "Challenges of Applied Cryoelectrics II" Weilburg, Germany, May 26-28, 2003.
7. " $1/f$ Noise in the Critical Current of Josephson Junctions: Implications for Flux Qubits," John Clarke, seminar, Department of Microtechnology and Nanoscience, Chalmers University of Technology, Gothenburg, Sweden, June 16, 2003.
8. "Progress in Flux Qubits," John Clarke, seminar, Department of Microtechnology and Nanoscience, Chalmers University of Technology, Gothenburg, Sweden, June 16, 2003.
9. "Use of RC-Shunted SQUIDs for Single-Shot Readout of Flux Qubits," T.L. Robertson, B.L.T. Plourde, S. Linzen, P.A. Reichardt, T. Hime, and J. Clarke, 9th International Superconductive Electronics Conference (ISEC 2003), Sydney, Australia, July 7-11, 2003.
10. "Decoherence in Superconducting Qubits from $1/f$ Noise," D.J. Van Harlingen, T.A. Crane, B.L.T. Plourde, T.L. Robertson P.A. Reichardt and John Clarke, invited talk by Dale Van Harlingen, Sixth European Conference on Applied Superconductivity (EUCAS), Sorrento, Italy, September 14-18, 2003.

11. "Backaction in SQUID-Readout of a Flux Qubit," John Clarke, invited talk at the International Conference on Solid State Quantum Information Processing, Amsterdam, The Netherlands, December 19, 2003
12. "Macroscopic Quantum Coherence In Superconducting Devices: Flux Qubits," Britton Plourde, Syracuse University, Physics Colloquium, February 3, 2004.
13. "The Ubiquitous Flux Vortex: SQUIDs, Qubits and Dynamics," Britton Plourde, Syracuse University, Condensed Matter Seminar, February 4, 2004.
14. "Quantum Coherence in Nanoscale Superconducting Devices: Flux Qubits," Britton Plourde, University of Massachusetts, Condensed Matter Seminar, February 17, 2004.
15. "Quantum Coherence In Nanoscale Superconducting Devices: Flux Qubits," Britton Plourde, University of Minnesota, Condensed Matter Seminar, February 19, 2004.
16. "Flux Qubits: Readout, Hot Electrons and Adjustable Coupling," John Clarke, invited talk, International Symposium on Mesoscopic Superconductivity and Spintronics, Atsugi, Japan, March 4, 2004.
17. "Spectroscopy of Large Inductance Flux Qubits," T.L. Robertson, T. Hime, S. Linzen, B.L.T. Plourde, P.A. Reichardt, C.E. Wu, F.K. Wilhelm and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
18. "Back-Action of RC-Shunted SQUID on Three-Junction Flux Qubit," T. Hime, S. Linzen, B.L.T. Plourde, P.A. Reichardt, T.L. Robertson, C.E. Wu, F.K. Wilhelm and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
19. "Variable Coupling Scheme for Entangling Flux Qubits," B.L.T. Plourde, J. Zhang, T.L. Robertson, T. Hime, S. Linzen, P.A. Reichardt, C.E. Wu, K.B. Whaley, F.K. Wilhelm and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
20. "Decoherence of Flux Qubits Due to Hot Quasiparticles in Readout SQUID," P.A. Reichardt, T. Hime, S. Linzen, B.L.T. Plourde, T.L. Robertson, C.E. Wu, F.K. Wilhelm and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
21. "Dephasing from $1/f$ Critical Current Fluctuations in Superconducting Qubits," T.A. Crane, D.J. Van Harlingen, T.L. Robertson, B.L.T. Plourde, P.A. Reichardt and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
22. "Measurements of the $1/f$ Noise in Josephson Junctions for Potential Use as Qubits," Chas Mugford, Jan Kycia, Matthias Korn, Michael Mueck and John Clarke, March Meeting of the American Physical Society, Montréal, Canada, March 22-26, 2004.
23. "Flux Qubits: Controllable Coupling, Adjustable Relaxation Rate and $1/f$ Noise," John Clarke, plenary invited talk, IV International Conference on Macroscopic Quantum Coherence and Computing, Napoli, Italy, June 7-10, 2004.
24. Conference Summary, John Clarke, invited closing address, IV International Conference on Macroscopic Quantum Coherence and Computing, Napoli, Italy, June 7-10, 2004.
25. "Large Inductance Flux Qubits: Coherent Manipulation and Controllable Coupling," John Clarke, Quantum Computing Program Review, Orlando, Florida, August 16-20, 2004.

26. "Back-Action of RC-Shunted SQUID on Three-Junction Flux Qubit," T. Hime, S. Linzen, B.L.T. Plourde, P.A. Reichardt, T.L. Robertson, C.-E. Wu, F.K. Wilhelm and John Clarke, Quantum Computing Program Review, Orlando, Florida, August 16-20, 2004.
27. "Superconducting flux qubits: readout, dynamics, and coupling," Britton L.T. Plourde, John Clarke, Travis Hime, Sven Linzen, Paul Reichardt, Tim Robertson, Birgitta Whaley, Frank Wilhelm, Cheng-En Wu, Jun Zhang, International Workshop on Solid State Based Quantum Information Processing, Herrsching, Bavaria, September 13-17, 2004.
28. "Superconducting flux qubits: readout, dynamics, and coupling," Britton L.T. Plourde, John Clarke, Travis Hime, Sven Linzen, Paul Reichardt, Tim Robertson, Birgitta Whaley, Frank Wilhelm, Cheng-En Wu, Jun Zhang, Quantum Information Science Seminar, Department of Physics, University of Illinois at Urbana-Champaign, September 29, 2004.
28. "Measurements of the $1/f$ Noise in Josephson Junctions for Potential Use as Qubits," Chas Mugford, Jan Kycia, Matthias Korn, Michael Mück and John Clarke, October 6, 2004, "Harnessing the Magic," Applied Superconductivity Conference, Jacksonville, Florida, October 3-8, 2004.
30. "Superconducting Flux Qubits: Coherent Oscillations, Controllable Coupling and $1/f$ Noise," John Clarke, seminar, University of Karlsruhe, Karlsruhe, Germany, November 10, 2004.
31. "Superconducting Flux Qubits: Coherent Oscillations and Controllable Coupling," John Clarke, seminar, Department of Physics, University of Copenhagen, Denmark, November 16, 2004.
32. "Superconducting Flux Qubits: Coherent Oscillations, Controllable Coupling and $1/f$ Noise," John Clarke, seminar, NEC Tsukuba, Japan, November 19, 2004.
33. "Large Inductance Flux Qubits: Coherent Manipulation and Controllable Coupling," John Clarke, invited talk, Wilhelm and Else Heraeus-Seminar, *Processing of Quantum Information in RSFQ Circuits and Qubits*, Bad Honnef, Germany, November 29, 2004.
34. "Flux Qubits and Readout Device with Two Independent Flux Lines," B.L.T. Plourde, T.L. Robertson, T. Hime, P.A. Reichardt, C.-E. Wu and John Clarke, March Meeting of the American Physical Society, Los Angeles, California, March 21-25, 2005.
35. "Quantum Coherence in a Superconducting Flux Qubit," T. Hime, B.L.T. Plourde, P.A. Reichardt, T.L. Robertson, C.-E. Wu and John Clarke, March Meeting of the American Physical Society, Los Angeles, California, March 21-25, 2005.
36. "Measurements of Dephasing in Superconducting Flux Qubits," C.-E. Wu, T. Hime, B.L.T. Plourde, P.A. Reichardt, T.L. Robertson and John Clarke, March Meeting of the American Physical Society, Los Angeles, California, March 21-25, 2005.
37. "Measurements of Relaxation in Superconducting Flux Qubits," P.A. Reichardt, T. Hime, B.L.T. Plourde, T.L. Robertson, C.-E. Wu and John Clarke, March Meeting of the American Physical Society, Los Angeles, California, March 21-25, 2005.
38. "Flux Qubits and Readout Device with Two Independent Flux Lines," B.L.T. Plourde, T.L. Robertson, T. Hime, S. Linzen, P.A. Reichardt, C.-E. Wu, John Clarke, K. Birgitta Whaley, J. Zhang, and Frank Wilhelm, "Solid State and Optics" seminar, Yale University, "April 6, 2005.
39. "Large-Inductance Superconducting Flux Qubits: Coherent Oscillations and Controllable Coupling", John Clarke, seminar, M.I.T., Cambridge, Massachusetts, April 11, 2005.

40. "Large-Inductance Superconducting Flux Qubits: Coherent Oscillations and Controllable Coupling," John Clarke, seminar, Physics Department, Michigan State University, East Lansing, Michigan, April 25, 2005.
41. "Large-Inductance Superconducting Flux Qubits: Coherent Oscillations and Controllable Coupling," John Clarke, seminar, Interdisciplinary Research Center for Superconductivity, University of Cambridge, England, May 3, 2005.
42. "Flux Qubits and Readout Device with Two Independent Flux Lines," B.L.T. Plourde, T.L. Robertson, T. Hime, S. Linzen, P.A. Reichardt, C.-E. Wu, John Clarke, K. Birgitta Whaley, J. Zhang, and Frank Wilhelm, seminar, Institute for Quantum Computing, University of Waterloo, IQC Seminar, May 9, 2005.
43. "Superconducting Flux Qubits; Quantum Coherence in a Macroscopic Circuit," John Clarke, invited talk, Science Day 2005, Lawrence Livermore National Laboratory, May 23, 2005.
44. "Novel Superconducting Devices to Isolate, Read Out, and Entangle Flux Qubits," John Clarke, *Quantum Computing Program Review* sponsored by the U.S. Army Research Office, National Security Agency, and Advanced Research & Development Activity, August 22-26, 2005, Tampa, Florida

Appendix B: Personnel

Principal Investigator:	John Clarke
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